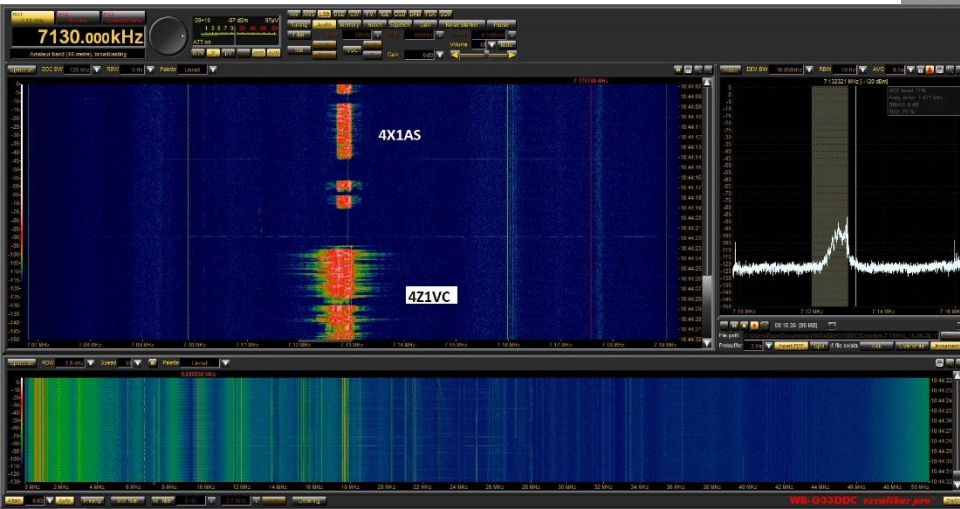


Beyond the horizon and further

The theory of HF propagation

Prof. Yosi Pinhasi

My homebrew solid-state transceiver



Communication channel



The electromagnetic spectrum

BAND	IEEE	FREQUENCY	WAVELENGTH	
Extremely Low Frequency	ELF	3 – 30Hz		
Super Low Frequency	SLF	30 – 300Hz		
Ultra Low Frequency	ULF	300 - 3,000 Hz	1,000 - 100 Km	
Very Low Frequency	VLF	3 - 30 KHz	100 – 10 Km	
Low Frequency	LF	30 - 300 KHz	10 - 1 Km	
Medium Frequency	MF	300 - 3,000 KHz	1 - 0.1 Km	
High Frequency	HF	3 - 30 MHz	100 - 10 m	
Very High Frequency	VHF	30 - 300 MHz	10 – 1 m	
Ultra High Frequency	UHF	300 - 3,000 MHz	1 - 0.1 m	
	L	1 - 2 GHz		
	S	2 - 4 GHz		
	Super High Frequency	SHF	3 - 30 GHz	10 - 1 cm
		C	4 - 8 GHz	
		X	8 - 12 GHz	
		Ku	12 - 18 GHz	
K		18 - 26.5 GHz		
	Ka	26.5 - 40 GHz		
Extremely High Frequency	EHF	30 - 300 GHz	1 - 0.1 cm	
	V	40 - 75 GHz		
	W	75 - 110 GHz		
Sub-millimeter (TeraHertz)	FIR	300 - 3,000 GHz	1 – 0.1 mm	
Mid infra-red	MIR	3 – 30 THz	100 – 10 μm	
Near infra-red	NIR	30 – 300 THz	10 – 1 μm	

Industrial, Scientific, Medical (ISM) frequencies



- 6.765-6.795 MHz (centre frequency 6.780 MHz)
- 13.553-13.567 MHz (centre frequency 13.560 MHz)
- 26.957-27.283 MHz (centre frequency 27.120 MHz)
- 40.66-40.70 MHz (centre frequency 40.68 MHz)
- 433.05-434.79 MHz (centre frequency 433.92 MHz) in Region 1
- 902-928 MHz (centre frequency 915 MHz) in Region 2
- 2.400-2.500 GHz (centre frequency 2.450 GHz)
- 5.725-5.875 GHz (centre frequency 5.800 GHz)
- 24-24.25 GHz (centre frequency 24.125 GHz)
- 61-61.5 GHz (centre frequency 61.25 GHz)
- 122-123 GHz (centre frequency 122.5 GHz)
- 5• 244-246 GHz (centre frequency 245 GHz)

Electromagnetic wave propagation phenomena

- בליעה (Absorption)
- החזרה (Reflection)
- שבירה (Refraction)
- עקיפה (Diffraction)
- פיזור (Scattering)
- התאבכות (Interference)
- נפיצה (Dispersion)

Electromagnetic waves propagation

$$\tilde{\mathbf{E}}(r, \theta, \phi) = \tilde{\mathbf{V}}(\theta, \phi) \cdot \frac{e^{-j\mathbf{k} \cdot \mathbf{r}}}{r}$$

Propagation factor:

$$k(f) = \frac{2\pi f}{c} \sqrt{\overbrace{\varepsilon_r(f)}^{n(f)}} = \frac{2\pi}{\lambda} \cdot n(f)$$

Refraction index: $n(f) = \sqrt{\varepsilon_r(f)} = \underbrace{n'(f)}_{\text{Real}} - j \underbrace{n''(f)}_{\text{Imaginary}}$

Complex propagation factor

$$\begin{aligned}k(f) &= \frac{2\pi f}{c} \cdot n(f) = \frac{2\pi f}{c} \cdot \underbrace{[n'(f) - j n''(f)]}_{n(f)} = \\ &= -j \underbrace{\frac{2\pi f}{c} \cdot n''(f)}_{\alpha(f)} + \underbrace{\frac{2\pi f}{c} \cdot n'(f)}_{\beta(f)} = -j\alpha(f) + \beta(f)\end{aligned}$$

$$\tilde{\mathbf{E}}_{out}(d) = \tilde{\mathbf{E}}_{in}(0) \cdot e^{-jk(f) \cdot d} = \tilde{\mathbf{E}}_{in}(0) \cdot e^{-j \overbrace{[-j\alpha(f) + \beta(f)]}^{k(f)} \cdot d} =$$

$$= \tilde{\mathbf{E}}_{in}(0) \cdot e^{-[\alpha(f) + j\beta(f)] \cdot d} = \tilde{\mathbf{E}}_{in}(0) \cdot e^{-\alpha(f) \cdot d} \cdot e^{-j\beta(f) \cdot d}$$

Power attenuation

$$P_{out} \propto \left| \tilde{\mathbf{E}}_{out}(d) \right|^2 = \tilde{\mathbf{E}}_{out}(d) \cdot \tilde{\mathbf{E}}_{out}^*(d) = \underbrace{\left| \tilde{\mathbf{E}}_{in}(0) \right|^2}_{\sim P_{in}} \cdot e^{-2\alpha(f) \cdot d}$$

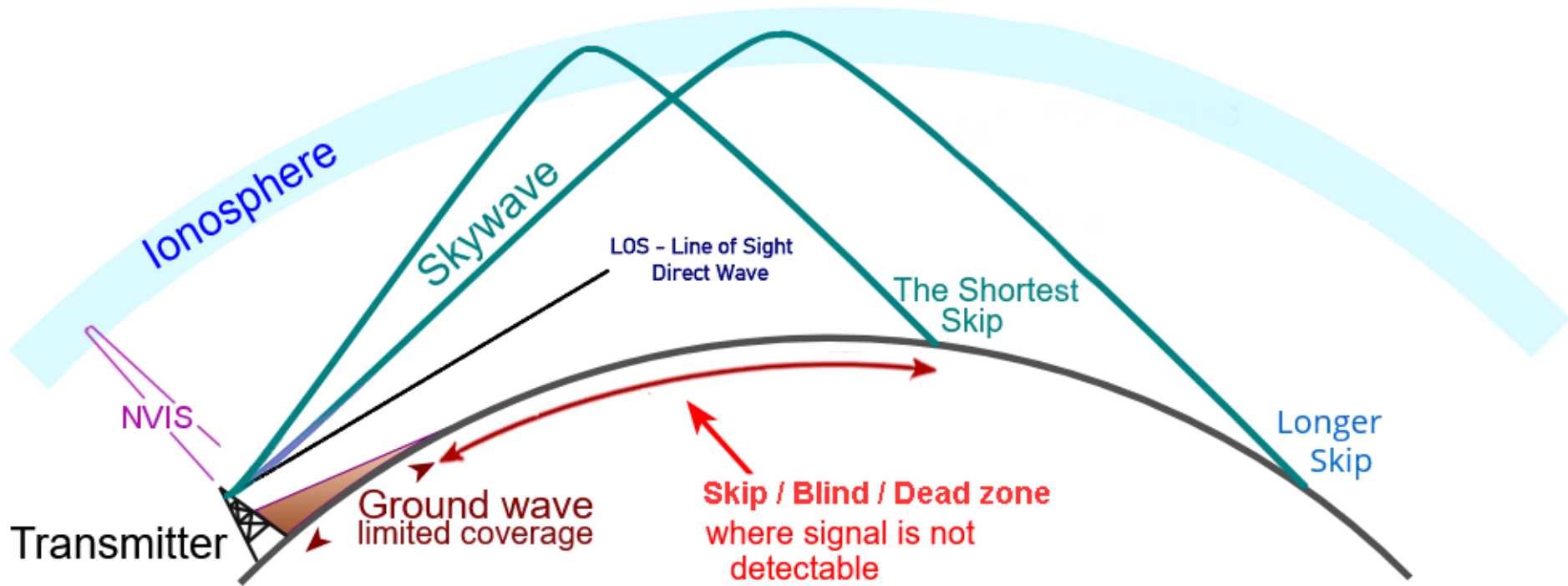
$$\frac{P_{out}}{P_{in}} = e^{-2\alpha(f) \cdot d}$$

$$\text{Attenuation [dB/m]} = 10 \cdot \log_{10} \left[e^{+2\alpha(f) \cdot 1} \right] =$$

$$= 20 \cdot \log_{10}(e) \cdot \alpha(f) = 20 \cdot \log_{10}(e) \cdot \underbrace{\left[\frac{2\pi f}{c} \cdot n''(f) \right]}_{\alpha(f)}$$

Propagation types

- Direct wave - Line of Sight – LOS
- Ground wave – Surface wave
- Sky wave
- Near Vertical Incidence (NVIS)



[Understanding HF Propagation: Forecast, Practice, Tutorial, Apps, Tools](#)

Understanding HF Skywave Propagation A Guide for Radio Hams

Doron Tal

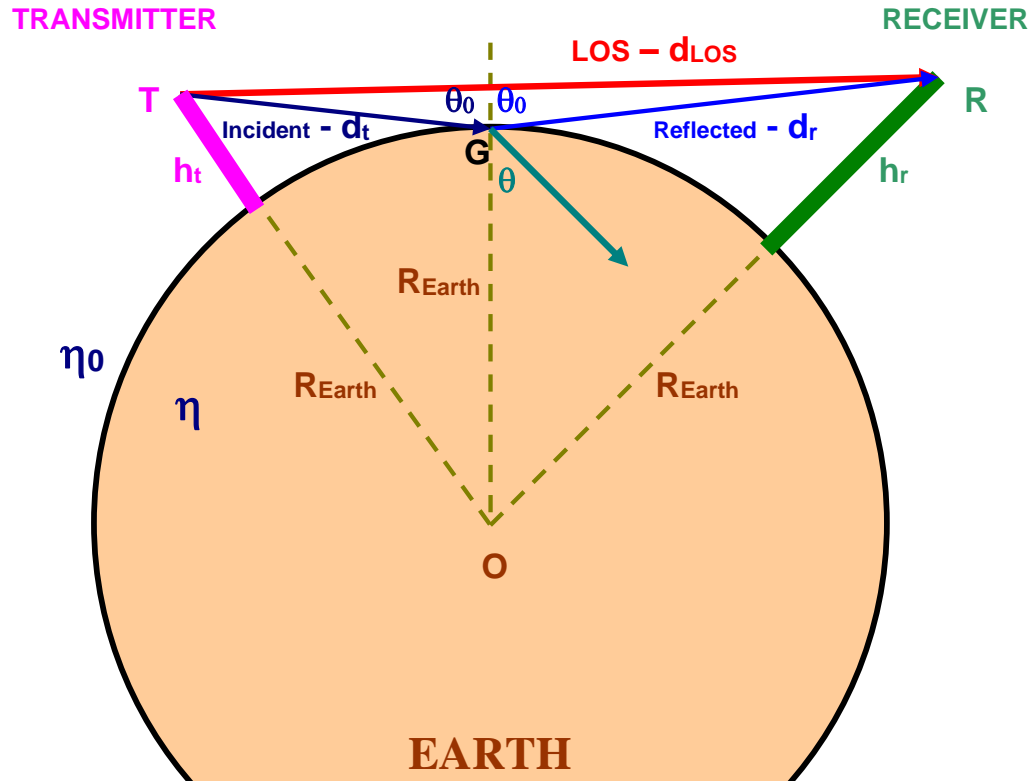
4X4XM

[Understanding HF Propagation: Forecast, Practice, Tutorial, Apps, Tools](https://www.qsl.net/4x4xm/HF-Propagation.htm#WNHFF)

<https://www.qsl.net/4x4xm/HF-Propagation.htm#WNHFF>

Direct wave - Line of Sight – LOS

Line of Sight - LOS



TGR:

$$d_{LOS}^2 = d_t^2 + d_r^2 - 2d_t d_r \cos(2\theta_0) = (d_t + d_r)^2 \left[1 - \frac{4d_t d_r}{(d_t + d_r)^2} \cos^2(\theta_0) \right]$$

Communication range

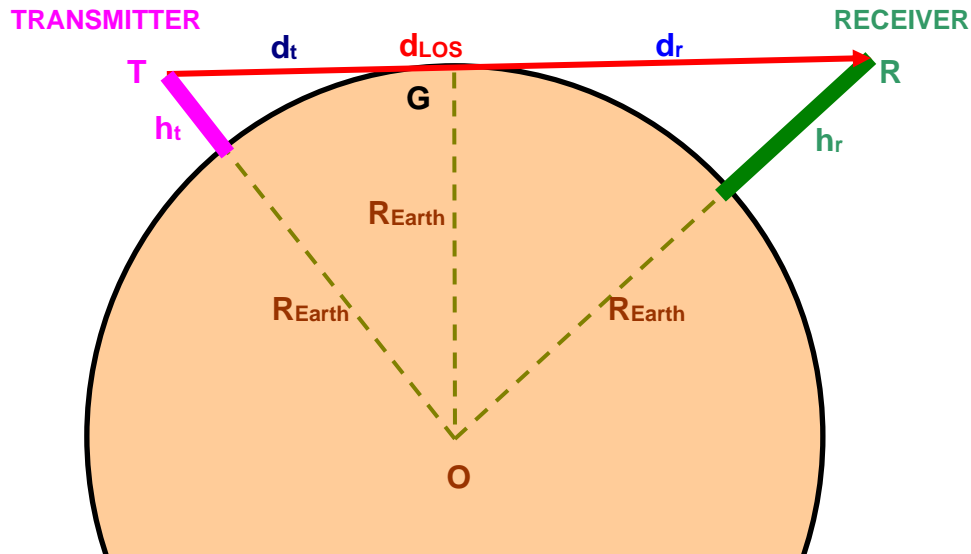
$$TGR: \quad d_{LOS}^2 = d_t^2 + d_r^2 - 2d_t d_r \cos(2\theta_0) = (d_t + d_r)^2 \left[1 - \frac{4d_t d_r}{(d_t + d_r)^2} \cos^2(\theta_0) \right]$$

$$TGO: \quad (h_t + R_{Earth})^2 = d_t^2 + R_{Earth}^2 + 2d_t R_{Earth} \cos(\theta_0)$$

$$RGO: \quad (h_r + R_{Earth})^2 = d_r^2 + R_{Earth}^2 + 2d_r R_{Earth} \cos(\theta_0)$$

$$R_{Earth} \cong 6,378 \text{ Km}$$

Horizon limit



$$\theta_0 = 90^\circ \longrightarrow d_t + d_r = \sqrt{(h_t + R_{Earth})^2 - R_{Earth}^2} + \sqrt{(h_r + R_{Earth})^2 - R_{Earth}^2} = d_{LOS_{max}}$$

$$h_t, h_r \ll R_{Earth} \longrightarrow d_{LOS_{max}} = \left(\sqrt{h_t} + \sqrt{h_r} \right) \cdot \sqrt{2R_{Earth}}$$

Ground wave

Ground wave

- Surface wave
- Below 5MHz
- Quickly attenuated
- Vertically polarized
- Tens of km – up to 100km

Propagation in the Atmosphere

The Atmosphere

- Nitrogen - N_2 - 78.084%
- Oxygen - O_2 - 20.948%
- Argon – Ar - 0.934%
- Carbon dioxide - CO_2 - 0.0314%
- Trace gases - 0.0434%
- Water vapor - H_2O - 1% at sea level

Atmospheric layers

70,000Km	Outer space				Magnetosphere
10,000Km				Plasmasphere	
1,000Km	Atmosphere	Exosphere	Ionosphere		
700Km		Thermosphere			
80Km		Mesosphere			
50Km		Stratosphere			
8-18Km		Troposphere			

The atmosphere of Earth



Refraction index, Refractivity

$$n(f) = \sqrt{\varepsilon_r(f)} = \sqrt{1 + \chi'_e(f) - j\chi''_e(f)} \cong$$

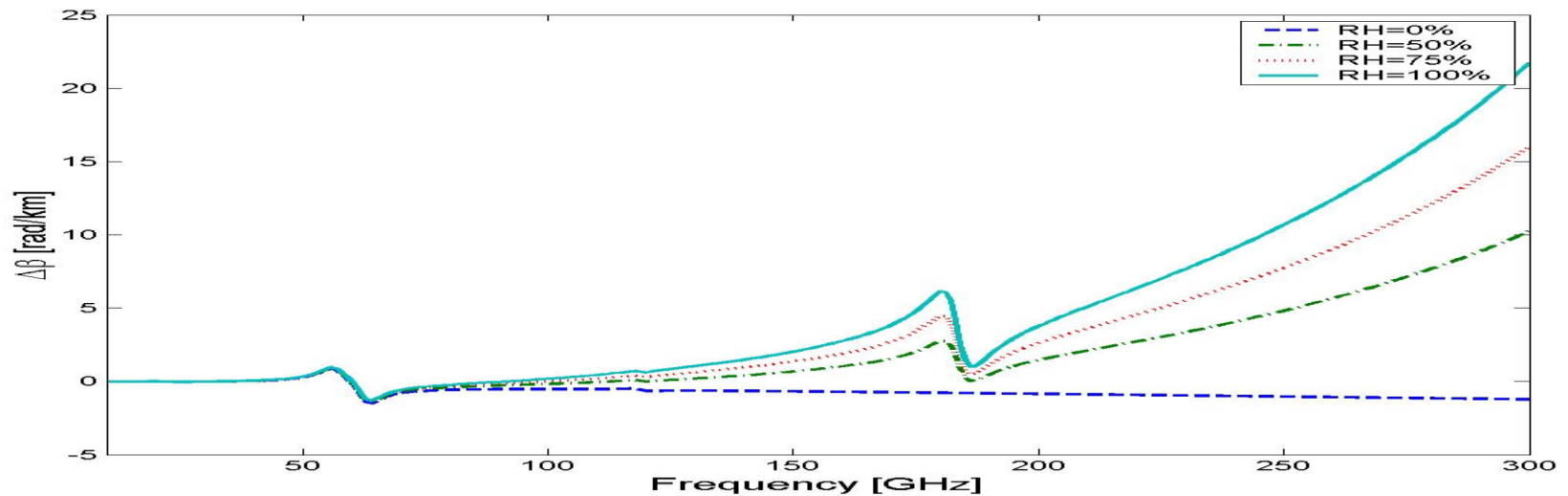
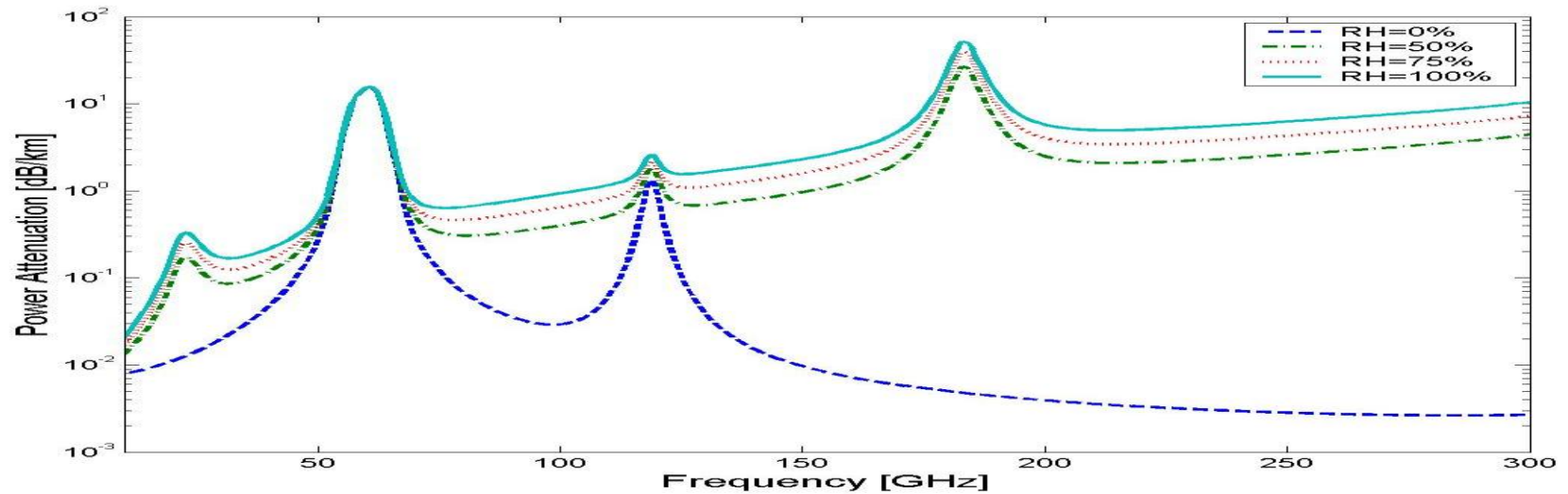
$$\cong 1 + \underbrace{\frac{1}{2}\chi'_e(f)}_{[N_0 + N'(f)] \cdot 10^{-6}} - j \underbrace{\frac{1}{2}\chi''_e(f)}_{N''(f) \cdot 10^{-6}} =$$

$$= \underbrace{1 + [N_0 + N'(f)] \cdot 10^{-6}}_{n'(f) - \text{Real}} - j \underbrace{N''(f) \cdot 10^{-6}}_{n''(f) - \text{Imaginary}}$$

Propagation factor

$$\begin{aligned}
 k(f) &= \frac{2\pi f}{c} \cdot n(f) = -j \underbrace{\frac{2\pi f}{c} \cdot N''(f) \cdot 10^{-6}}_{\alpha(f) = \frac{\pi f}{c} \chi_e''(f)} + \\
 &+ \underbrace{\frac{2\pi f}{c} \cdot (1 + N_0 \cdot 10^{-6}) + \frac{2\pi f}{c} \cdot N'(f) \cdot 10^{-6}}_{\beta(f) = \frac{2\pi f}{c} + \frac{\pi f}{c} \chi_e'(f)} + \overbrace{\frac{2\pi f}{c} \cdot N'(f) \cdot 10^{-6}}^{\Delta\beta(f)}
 \end{aligned}$$

בליעה באטמוספירה בתחום ה-EHF



The Ionosphere

Regions

- D layer – Day: 50-80 km – NO^+
- E layer – Day & Night: 90-120 km – O_2^+ , NO^+
- Sporadic E – Summer days: 90-120 km – O_2^+ , NO^+
- F1 layer – Day - 130 to 210 km - O_2^+ , NO^+ , O^+
- F2 layer – Day - over 210 km - O^+ , N^+ , H^+

GEOCORONA 15.5 Earth radii

*** depending on solar activity

500-1.000 km 1.700.000-3.300.000 ft *** +1.500°C
EXOSPHERE

350-800 km 1.100.000-2.600.000 ft *** +1.500°C Thermopause

IONOSPHERE
F2
F1
E
D

220-800 km

150-220 km

90-120 km

60-90 km

THERMOSPHERE

Kármán line
100 km 330.000 ft

80-85 km 260.000-280.000 ft -100°C

MESOSPHERE

Mesopause

51 km 170.000 ft -3°C

Stratopause

Ozone Layer
15-35 km 49.000-110.000 ft

STRATOSPHERE

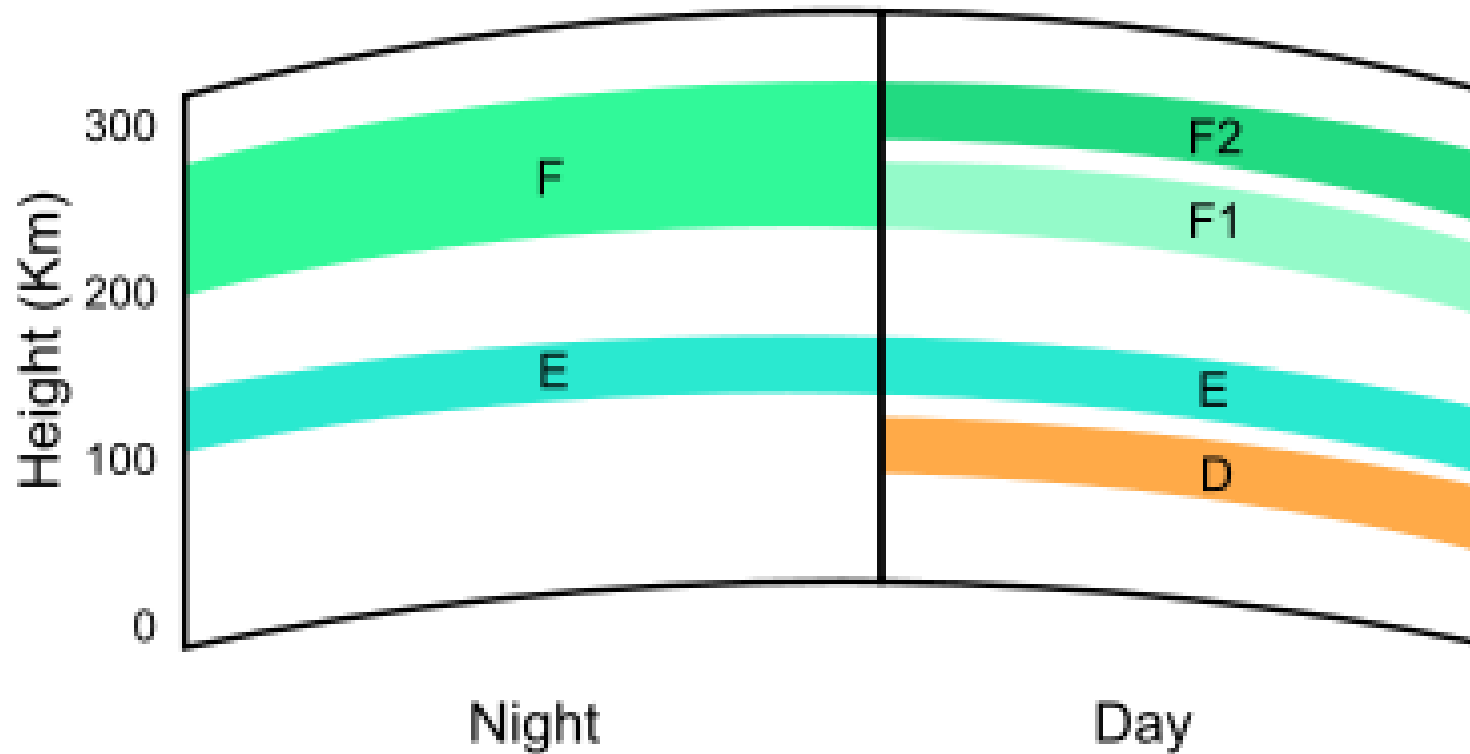
9-17 km 30.000-56.000 ft -55°C to -75°C

TROPOSPHERE

Tropopause

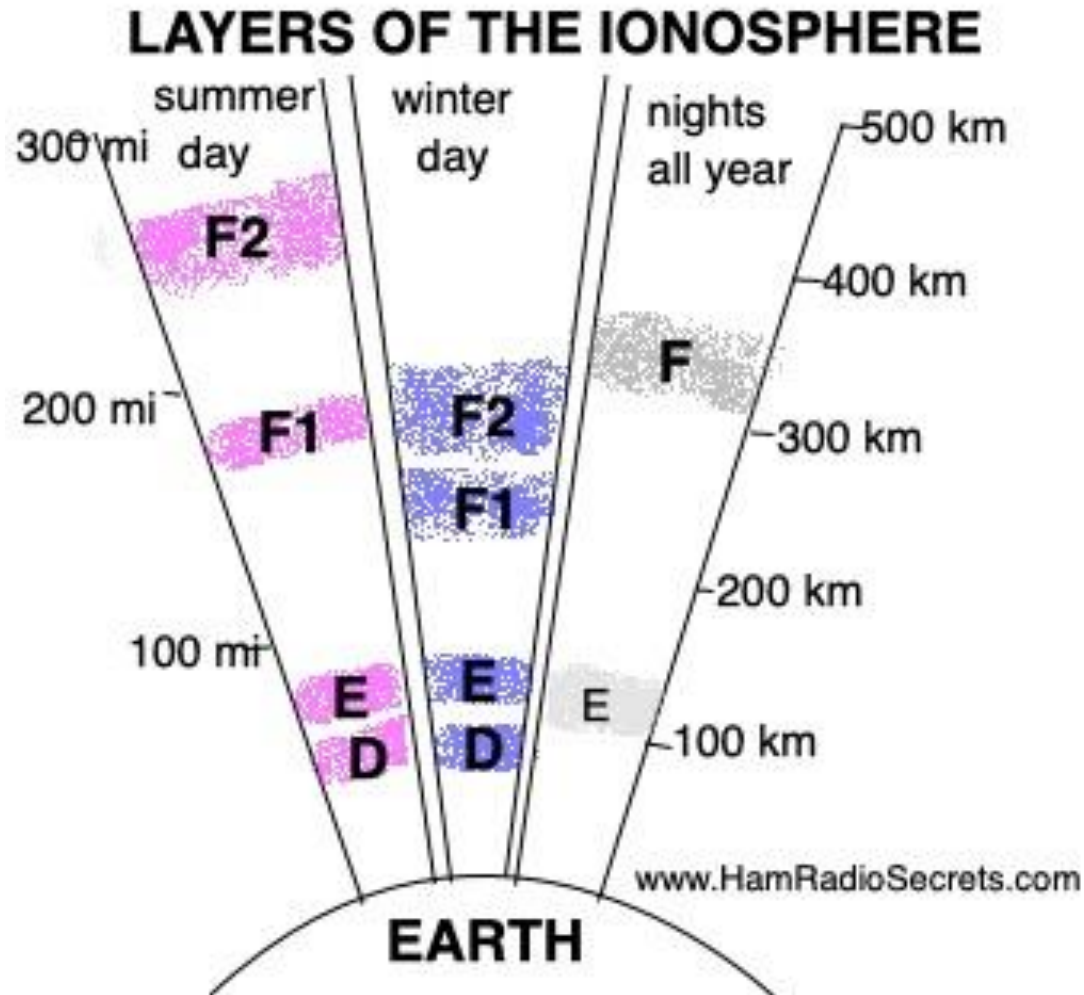
EARTH

Characteristics of different ionospheric layers – Buzztech



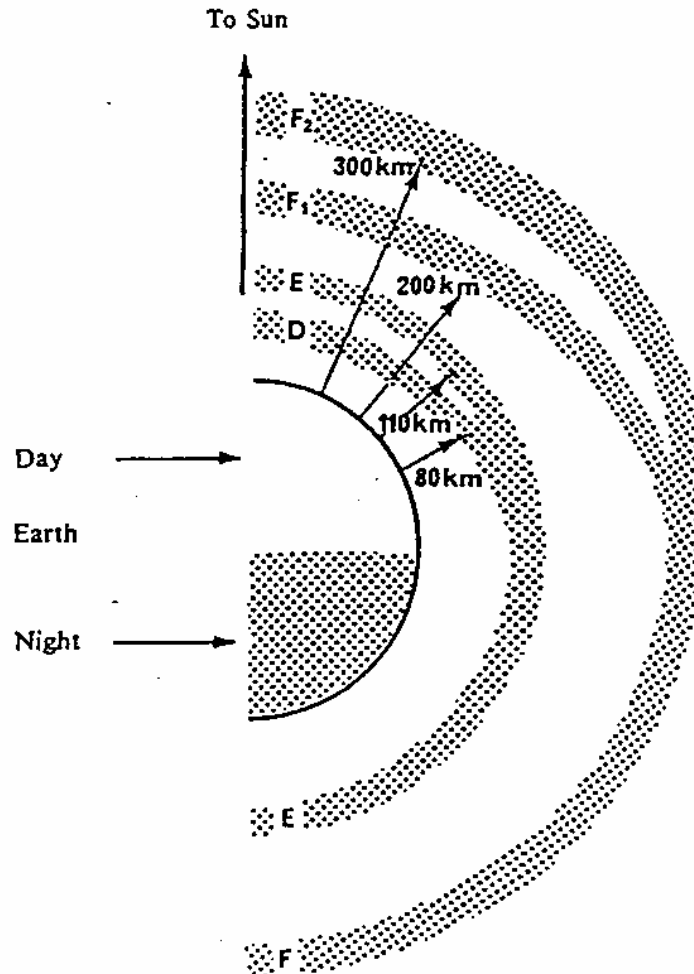
Ionospheric Propagation of Radio Waves Gives Ham Radio Operators "Seven League Boots"!

<https://www.hamradiosecrets.com/ionospheric-propagation-of-radio-waves.html>



THE IONOSPHERE AND ITS EFFECTS ON RADIOWAVE PROPAGATION

A guide with background to ITU-R procedures for radio
planners and users



D - Region

- Mesosphere
- Only daytime
- Altitude: 50 to 80 km
- Average altitude: 70km
- Thickness: 10 km
- NO^+ excited by UVC (121.6nm)
- Low electron density 10^8 - 10^{10} electrons/ m^3
- High recombination rate 10^6 1/sec

D region – interaction with EM waves

- Reflection of LF, VLF, ELF
- $f_e = 1.2 \cdot 10^6$ collisions per second
- Absorbing 160m, 80m, 60m, 40m
- Attenuation of MF and HF below 10MHz
(at noon)
- Defines the Lowest Usable Frequency –
LUF
- Solar bursts of X-rays (0.1–1nm) causing
33 blackout (minutes to hours)

E - Region

- Thermosphere
- Only daytime
- 90 to 120 km
- Average altitude: 110km
- Thickness: 25 km
- O_2^+ , NO^+ excited by EUV (1-10nm)
- Maximum electron density $4 \cdot 10^{11}$ electrons/m³
- High recombination rate at evening
- Disappears within couple of hours after sunset
- Reflection of frequencies below 10MHz
- 34 • Transmitted near vertically (NVIS)

Sporadic –Es – plasma clouds

- Thermosphere
- Only daytime
- Summer days
- 90 to 120 km
- Thin layer
- O_2^+ , NO^+
- Temporary high ionization regions (Plasma clouds)
- Electron density above 10^{11} electrons/m³
- Reflection of frequencies up to 150MHz

F1 - Layer

- Thermosphere
- Only daytime
- Mainly summer days
- 130km to 210km (180km)
- Thickness: 20 km
- O^+ , O_2^+ , NO^+ excited by EUV (10-100nm)
- Electron density $2 \cdot 10^{11}$ electrons/m³ at 180km
- High recombination rate at evening
- Reflection of frequencies below tens of MHz
- Summer, often reflects HF at the 30m, 20 bands

F2 - Layer

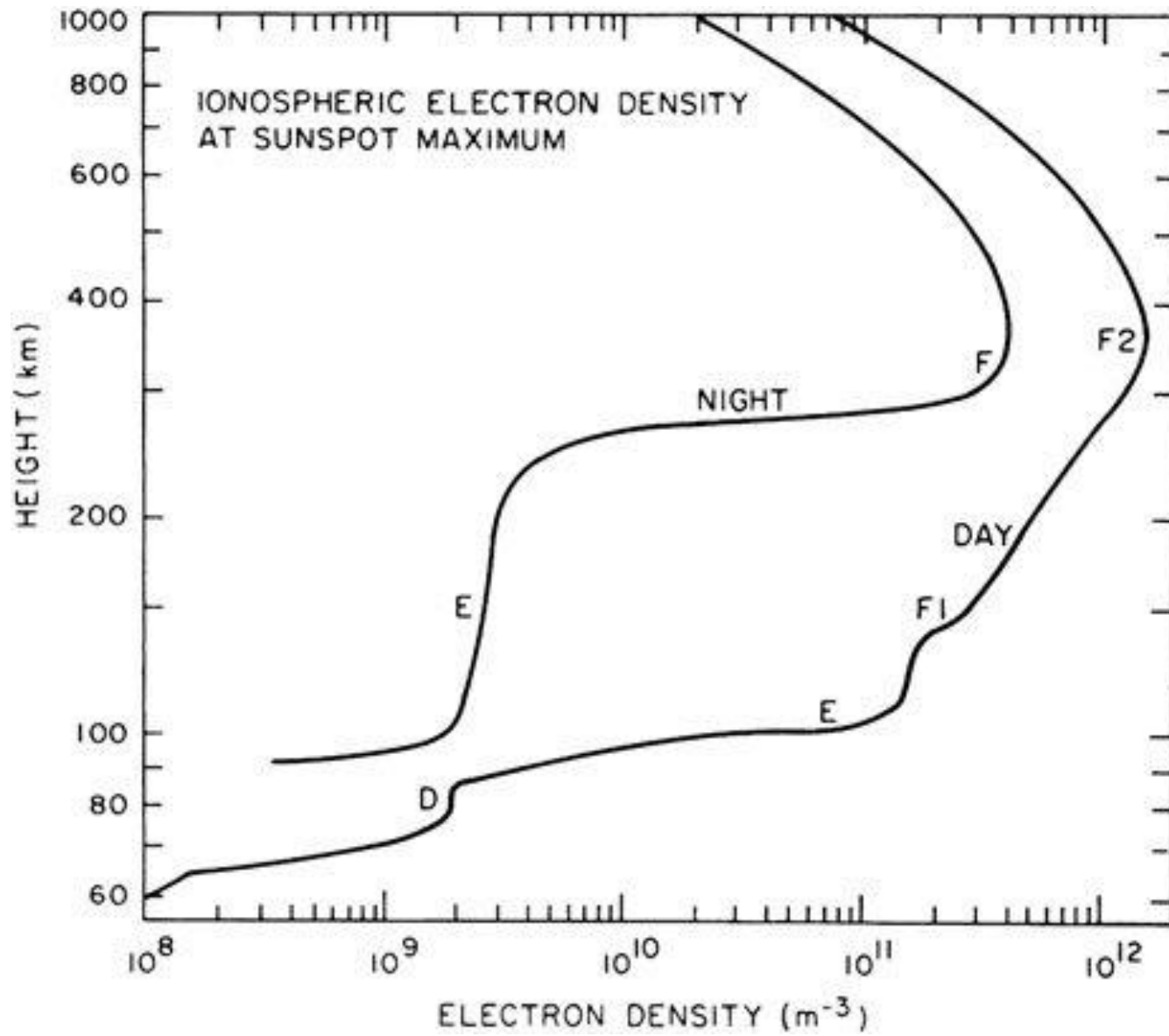
- Thermosphere
 - Only daytime
 - Summer days
 - 250km to 500km
 - Average altitude: 325km
 - Thickness: 200 km
 - O⁺ excited by EUV (10-100nm)
 - Maximum electron density $1 \cdot 10^{12}$ - $4 \cdot 10^{12}$ electrons/m³
 - Average: $2 \cdot 10^{12}$ m⁻³
 - High recombination rate at evening
- 37 Reflection of frequencies below tens of MHz

F – Region during the night

- F1 merges into F2
- Thermosphere
- Night
- All seasons
- 350-400 km
- O_2^+ , NO^+ , O^+ excited by EUV (10-100nm)
- Electron density reduces to $5 \cdot 10^{10}$ electrons/m³
- Reflection of frequencies below tens of MHz

Electron density – m^{-3}

				DAY	NIGHT
1,000Km	Atmosphere	Exosphere	Ionosphere		
700Km		Thermosphere		F2 ($O^+, N^+ 10^{12}$)	F ($O^+ 5 \cdot 10^{10}$)
210Km					
200Km				F1 ($O^+ 2 \cdot 10^{11}$)	
140Km					
120Km		E ($O_2^+ 10^{11}$)		E ($O_2^+ 1.5 \cdot 10^9$)	
90Km					
80Km		Mesosphere		D ($NO^+ 10^8-10^{10}$)	
60Km					
50Km		Stratosphere			
8-18Km	Troposphere				



Plasma density (electron/cm³)

10⁴

10⁵

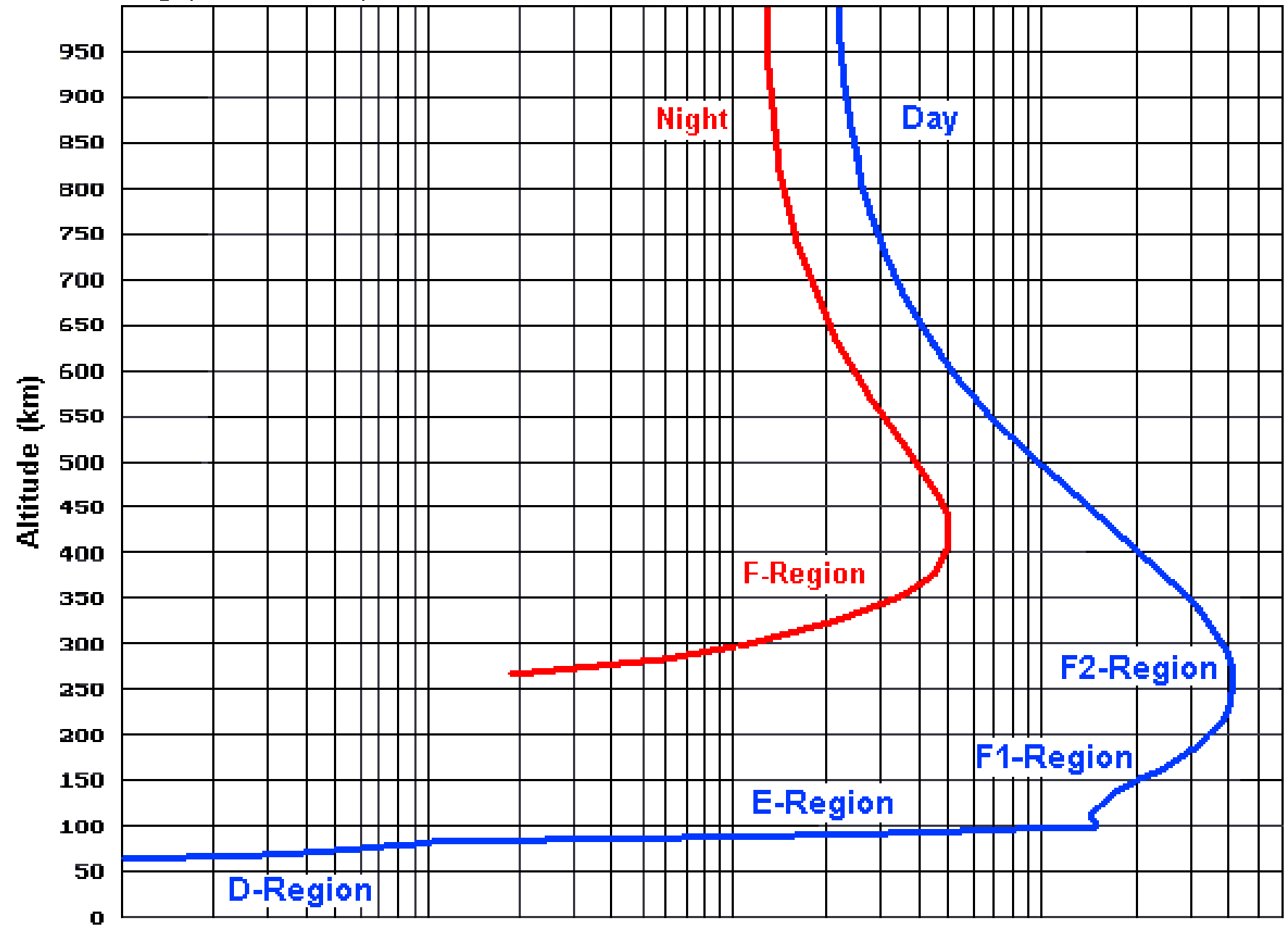
10⁶

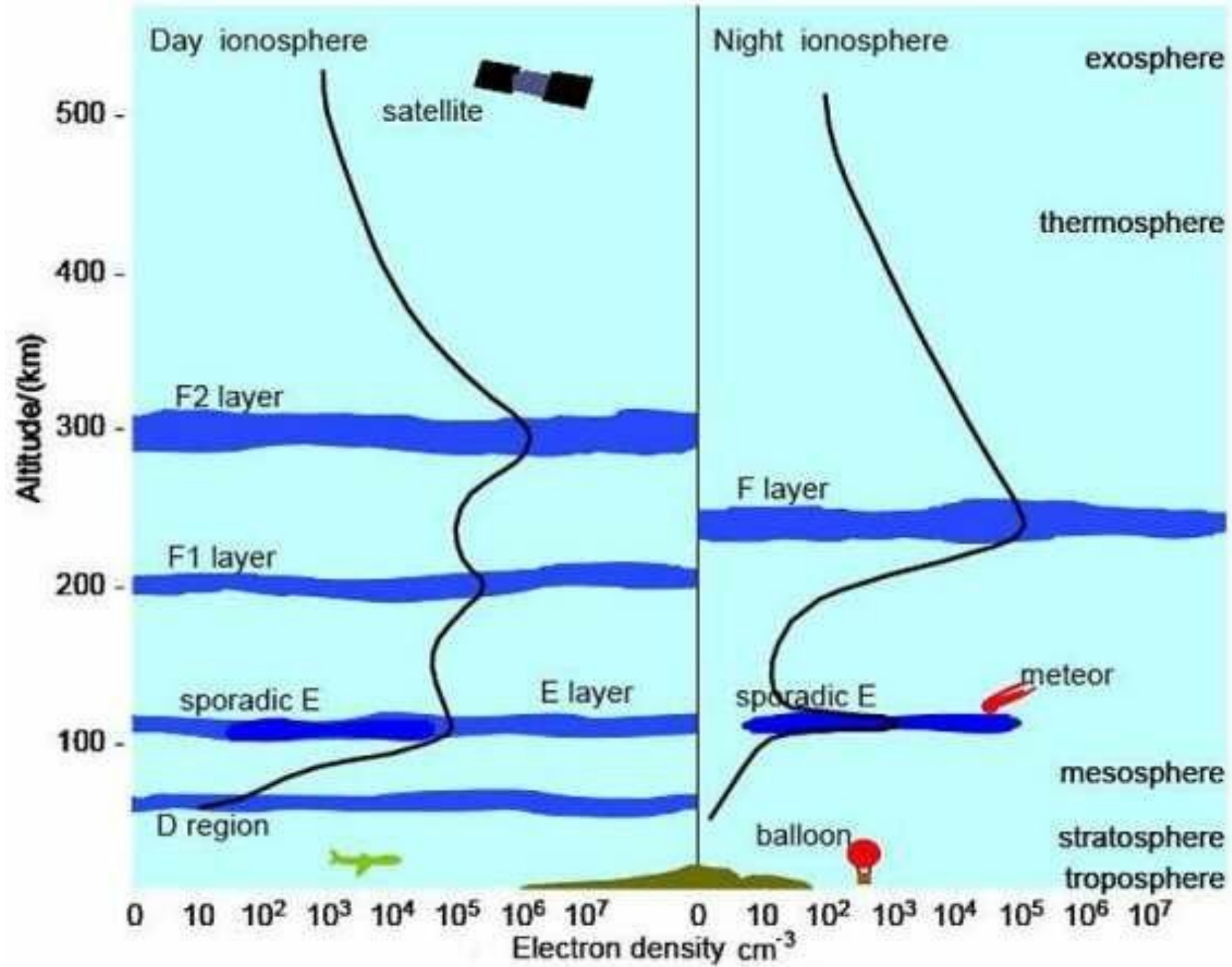
Plasma density (electron/m³)

10¹⁰

10¹¹

10¹²





Plasma

Dielectric coefficient

$$\varepsilon_r(f) = 1 - \frac{\left(\frac{f_p}{f}\right)^2}{1 - j\frac{f_e}{f}} = \underbrace{\left[1 - \frac{\left(\frac{f_p}{f}\right)^2}{1 + \left(\frac{f_e}{f}\right)^2}\right]}_{\varepsilon'(f)} - j \underbrace{\left[\frac{\left(\frac{f_p}{f}\right)^2}{1 + \left(\frac{f_e}{f}\right)^2}\right]}_{\varepsilon''(f)} \cdot \frac{f_e}{f}$$

$$f_p = \frac{1}{2\pi} \sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}} = 8.977 \sqrt{N_e}$$

$$e \cong 1.6 \cdot 10^{-19} \text{ C} \quad m_e = 9.11 \cdot 10^{-31} \text{ Kg} \quad \varepsilon_0 = 8.85 \cdot 10^{-12} \text{ F / m}$$

Attenuation and Wavenumber

- Attenuation:

$$\alpha(f) = -\text{Im}[k(f)] = \frac{2\pi f}{c} \sqrt{\frac{\varepsilon'(f)}{2}} \left[\sqrt{1 + \left(\frac{\varepsilon''(f)}{\varepsilon'(f)}\right)^2} - 1 \right]$$

- Wavenumber:

$$\beta(f) = \text{Re}[k(f)] = \frac{2\pi f}{c} \sqrt{\frac{\varepsilon'(f)}{2}} \left[\sqrt{1 + \left(\frac{\varepsilon''(f)}{\varepsilon'(f)}\right)^2} + 1 \right]$$

Dielectric coefficient

$$f \gg f_e \longrightarrow \varepsilon_r(f) = 1 - \frac{\left(\frac{f_p}{f}\right)^2}{1 - j\frac{f_e}{f}} \approx 1 - \left(\frac{f_p}{f}\right)^2 = \varepsilon'(f)$$

$$f_p = \frac{1}{2\pi} \sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}} = 8.977 \sqrt{N_e}$$

$$e \cong 1.6 \cdot 10^{-19} \text{ C} \quad m_e = 9.11 \cdot 10^{-31} \text{ Kg} \quad \varepsilon_0 = 8.85 \cdot 10^{-12} \text{ F / m}$$

Propagation coefficient

Refraction index:

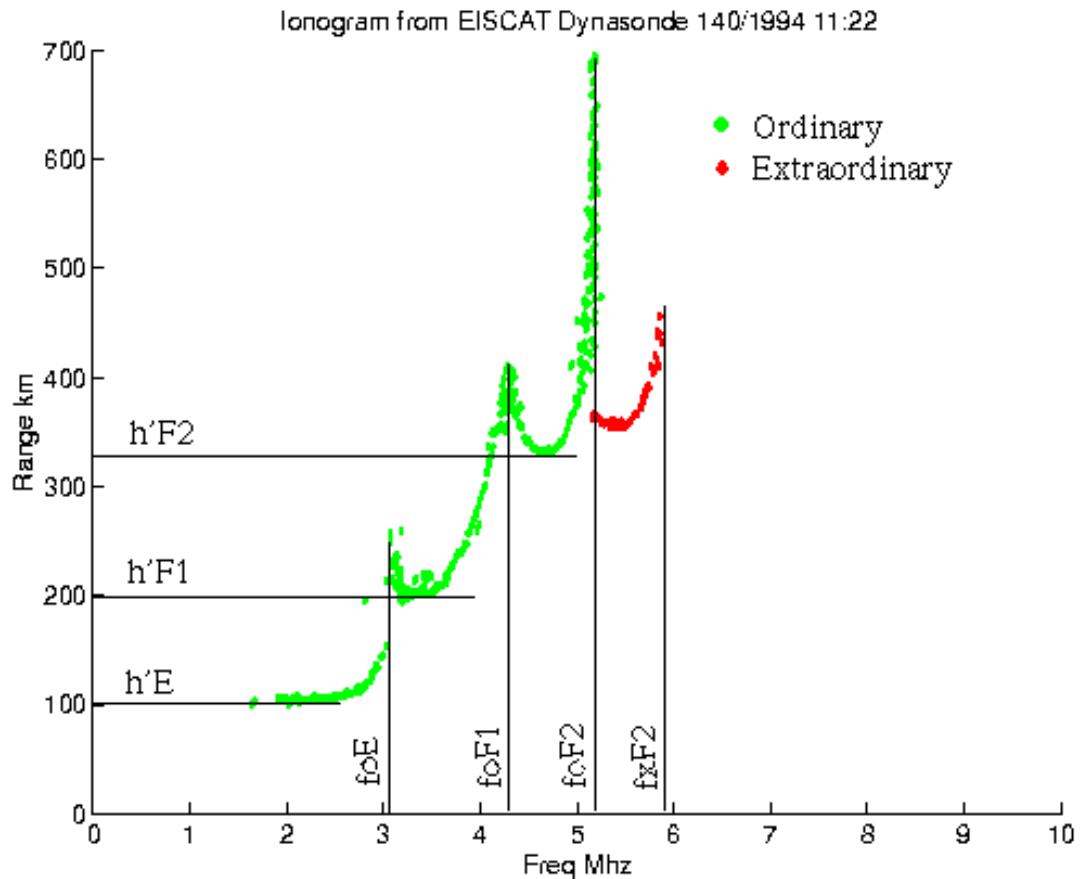
$$n(f) \approx \sqrt{\varepsilon'(f)} = \sqrt{1 - \left(\frac{f_p}{f}\right)^2}$$

Wavenumber:

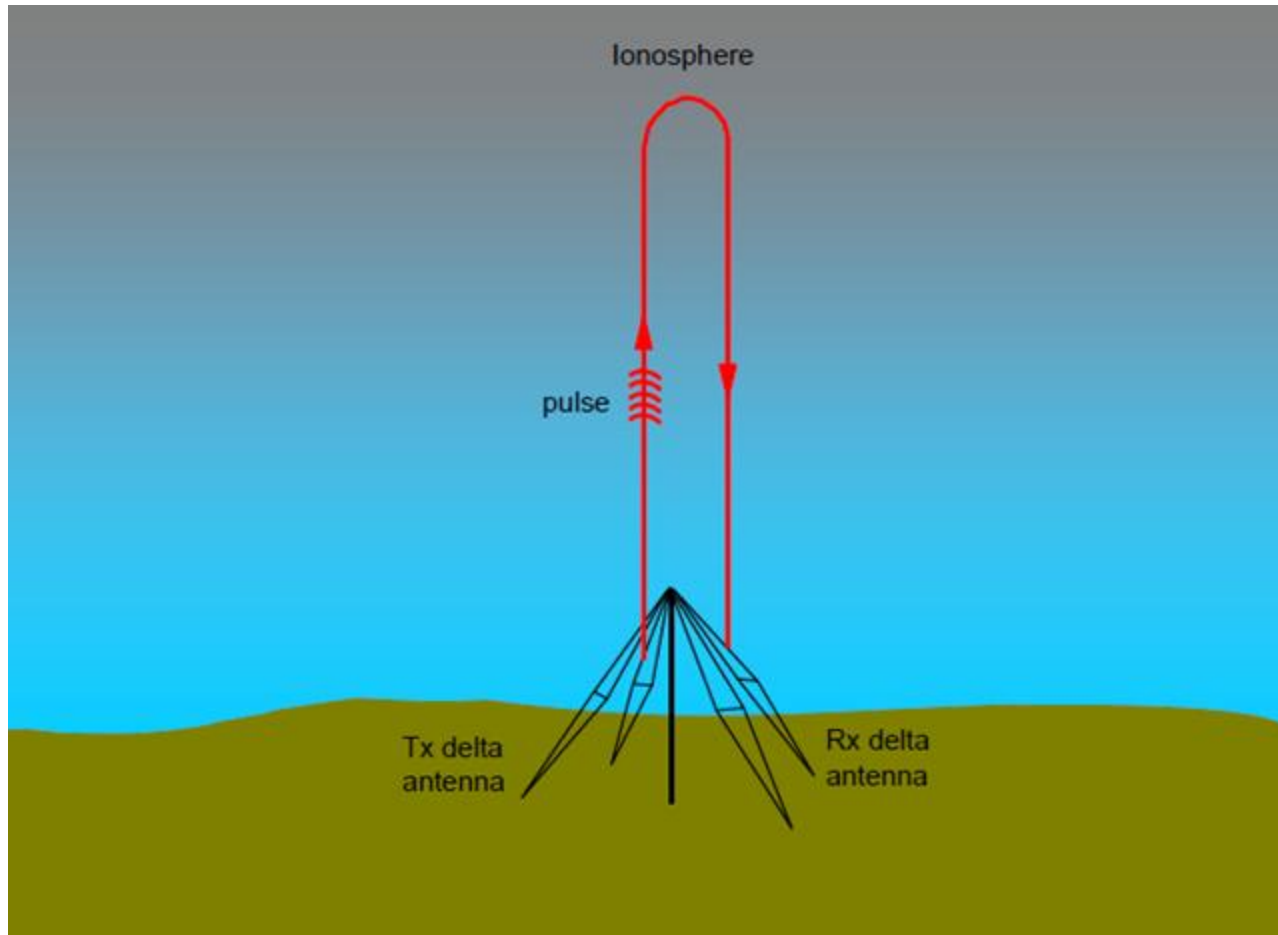
$$\beta(f) = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon'(f)} = \frac{2\pi}{c} \cdot \sqrt{f^2 - f_p^2}$$

Plasma (critical) ionospheric frequency

$$f_p = 8.977 \sqrt{N_e}$$

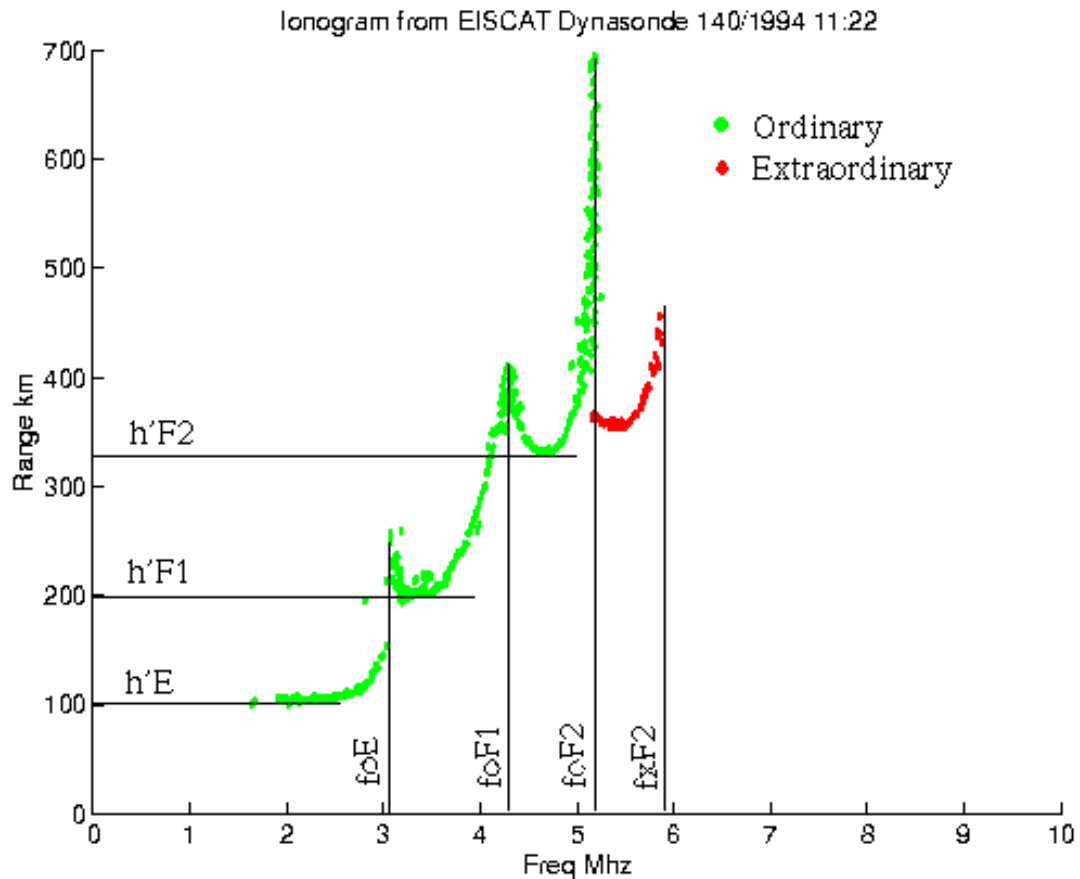


Ionosonde, chirp-sounder

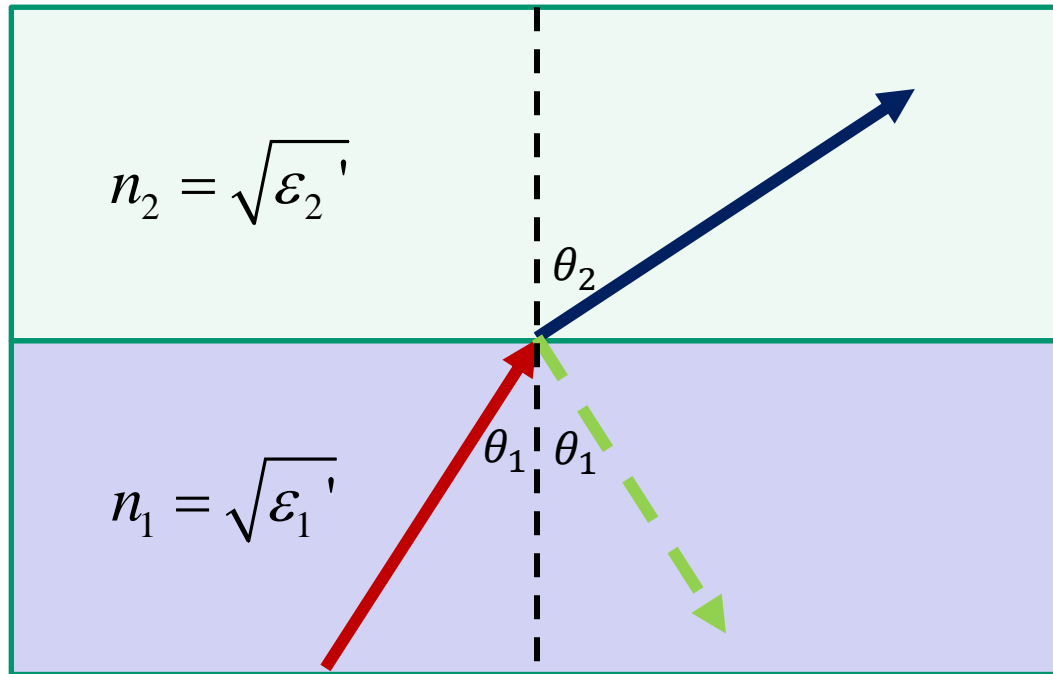


Critical ionospheric frequency measurement

$$f_p = 8.977 \sqrt{N_e}$$



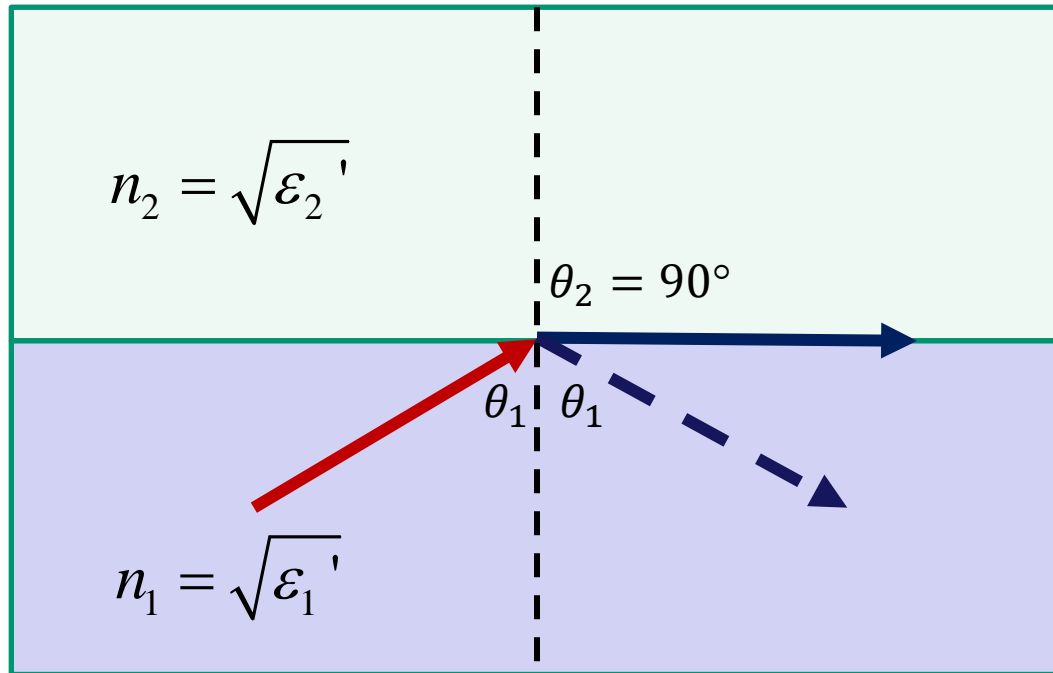
Oblique Refraction



Snell's law:
$$\sqrt{\epsilon_1'} \cdot \sin(\theta_1) = \sqrt{\epsilon_2'} \cdot \sin(\theta_2) \longrightarrow \sin(\theta_2) = \frac{n_1}{n_2} \cdot \sin(\theta_1) = \sqrt{\frac{\epsilon_1'}{\epsilon_2'}} \cdot \sin(\theta_1)$$

$$\sqrt{\epsilon_1'} > \sqrt{\epsilon_2'} \longrightarrow n_1 > n_2 \longrightarrow \theta_2 > \theta_1$$

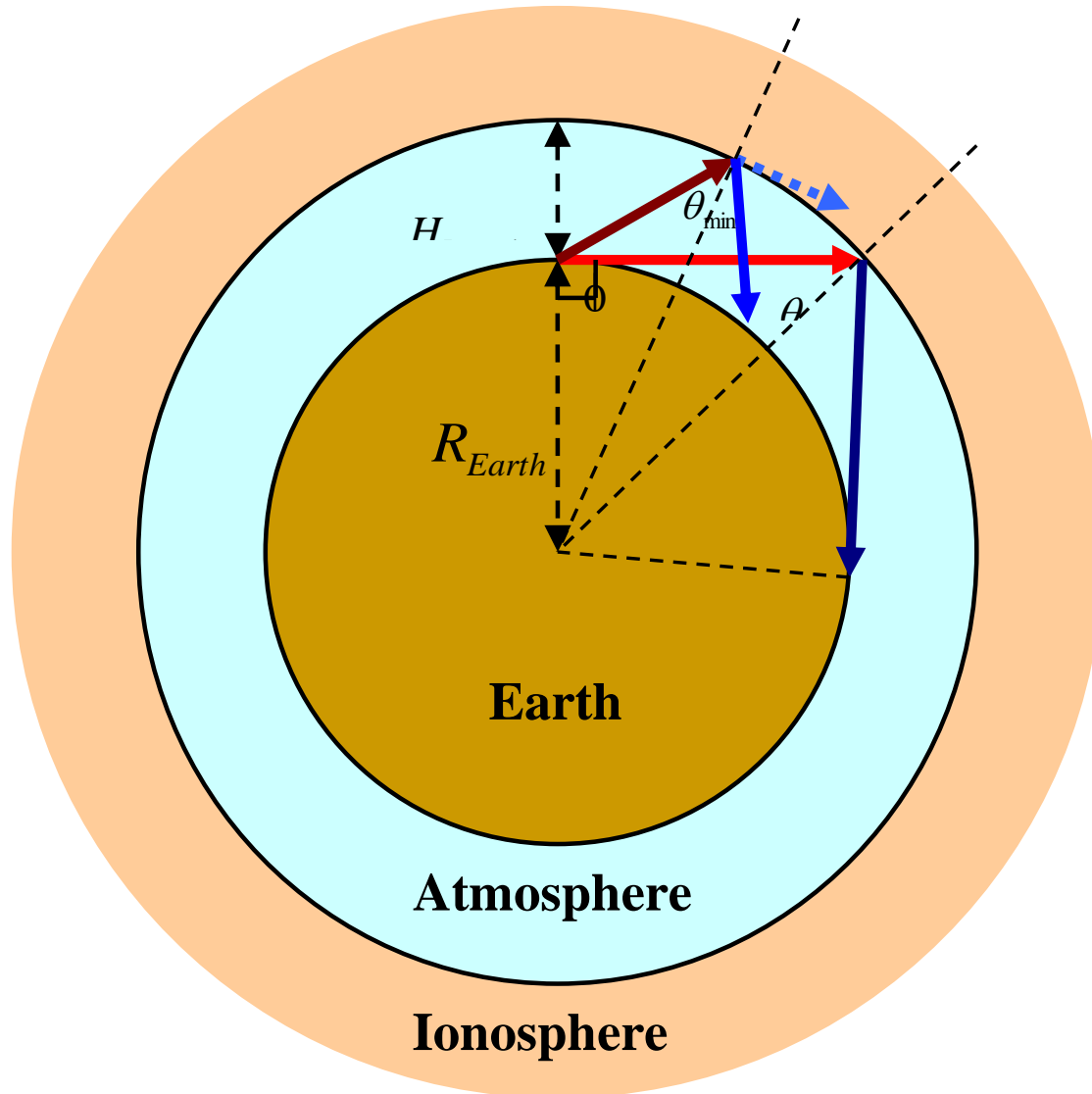
Total Reflection



$$\text{Total reflection: } \sqrt{\epsilon_1'} \cdot \sin(\theta_1) = \underbrace{\sqrt{\epsilon_2'} \cdot \sin(90^\circ)}_1 \longrightarrow \sin(\theta_{1_{\min}}) = \sqrt{\frac{\epsilon_2'}{\epsilon_1'}}$$

$$n_1 = \sqrt{\epsilon_1'} = 1 \longrightarrow \sin(\theta_{1_{\min}}) = n_2 = \sqrt{\epsilon_2'} < 1$$

Reflection from the Ionosphere



Minimum incidence angle for total reflection

Snell's law:

$$1 \cdot \sin(\theta_{\min}) = n(f) \cdot \sin(90^\circ) \longrightarrow \sin(\theta_{\min}) = n(f) = \sqrt{1 - \left(\frac{f_p}{f}\right)^2}$$

$$\theta > \theta_{\min} \longrightarrow \sin(\theta) > \underbrace{\sqrt{1 - \left(\frac{f_p}{f}\right)^2}}_{\sin(\theta_{\min})} \longrightarrow \underbrace{\sqrt{1 - \sin^2(\theta)}}_{\cos(\theta)} < \frac{f_p}{f} \longrightarrow f_p < f < \frac{f_p}{\cos(\theta)}$$

Shorter range: $l \downarrow \longrightarrow \theta \downarrow \longrightarrow \cos(\theta) \uparrow \longrightarrow f < \frac{f_p}{\cos(\theta)} \downarrow$

Maximum incidence angle

Maximum Usable Frequency

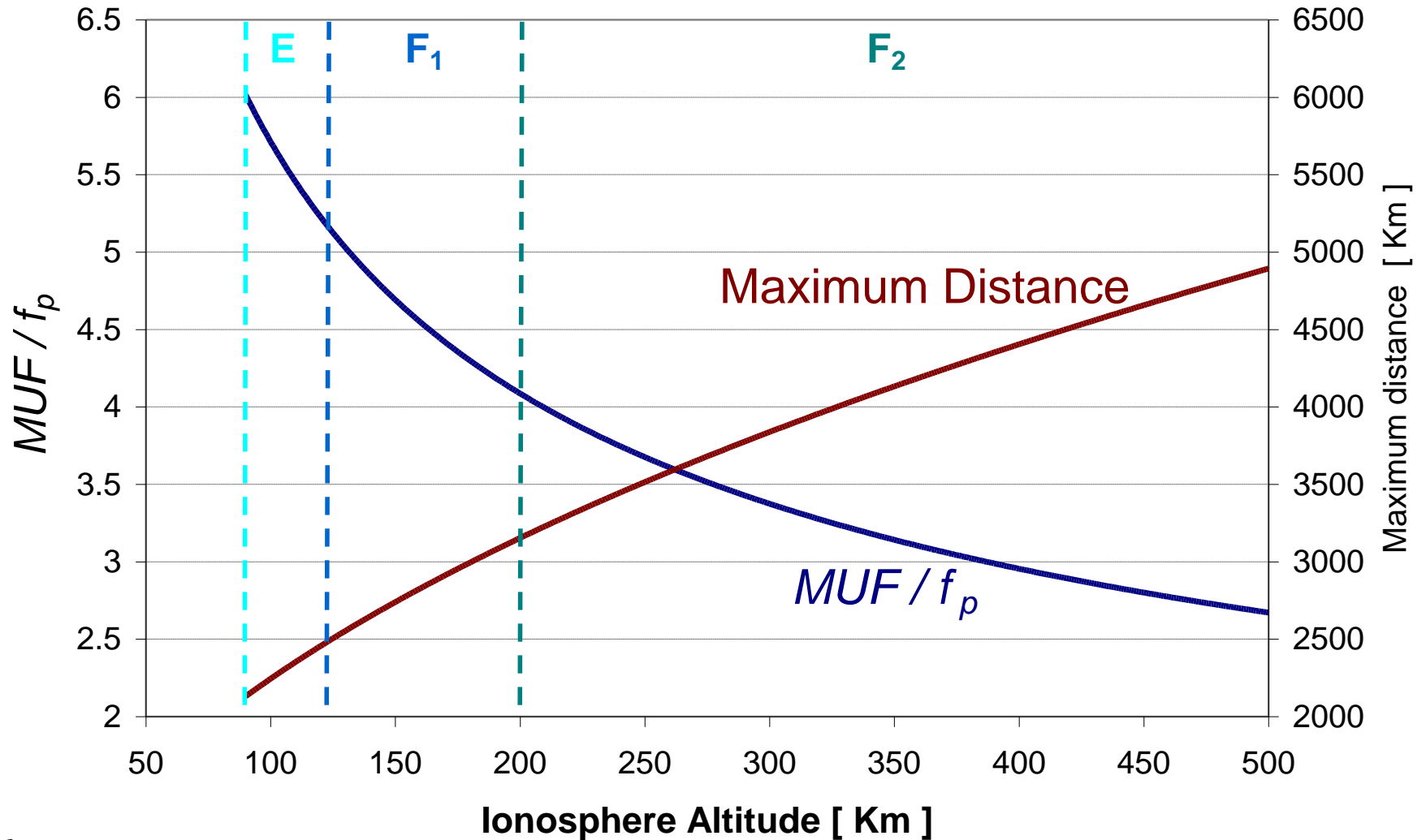
$$\sin(\theta_{\max}) = \frac{R_{\text{Earth}}}{R_{\text{Earth}} + H_{\text{Ionosphere}}}$$

$$MUF = \frac{f_p}{\cos(\theta_{\max})} = \frac{f_p}{\sqrt{1 - \sin^2(\theta_{\max})}} = \frac{f_p}{\sqrt{1 - \left(\frac{R_{\text{Earth}}}{R_{\text{Earth}} + H_{\text{Ionosphere}}} \right)^2}}$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow \cos(\theta_{\max}) \uparrow \longrightarrow MUF \downarrow$$

Above the MUF – no reflection from the Ionosphere !

MUF vs. Altitude



Important notes !

$$f_p \uparrow \longrightarrow MUF = \frac{f_p}{\cos(\theta_{\max})} \uparrow$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow \cos(\theta_{\max}) \uparrow \longrightarrow MUF \downarrow$$

Lowest Usable Frequency

- Below the LUF – high absorption in the D region !
- Only during day.
- Rises during solar flare bursts.
- May creates blackouts for minutes to hours.

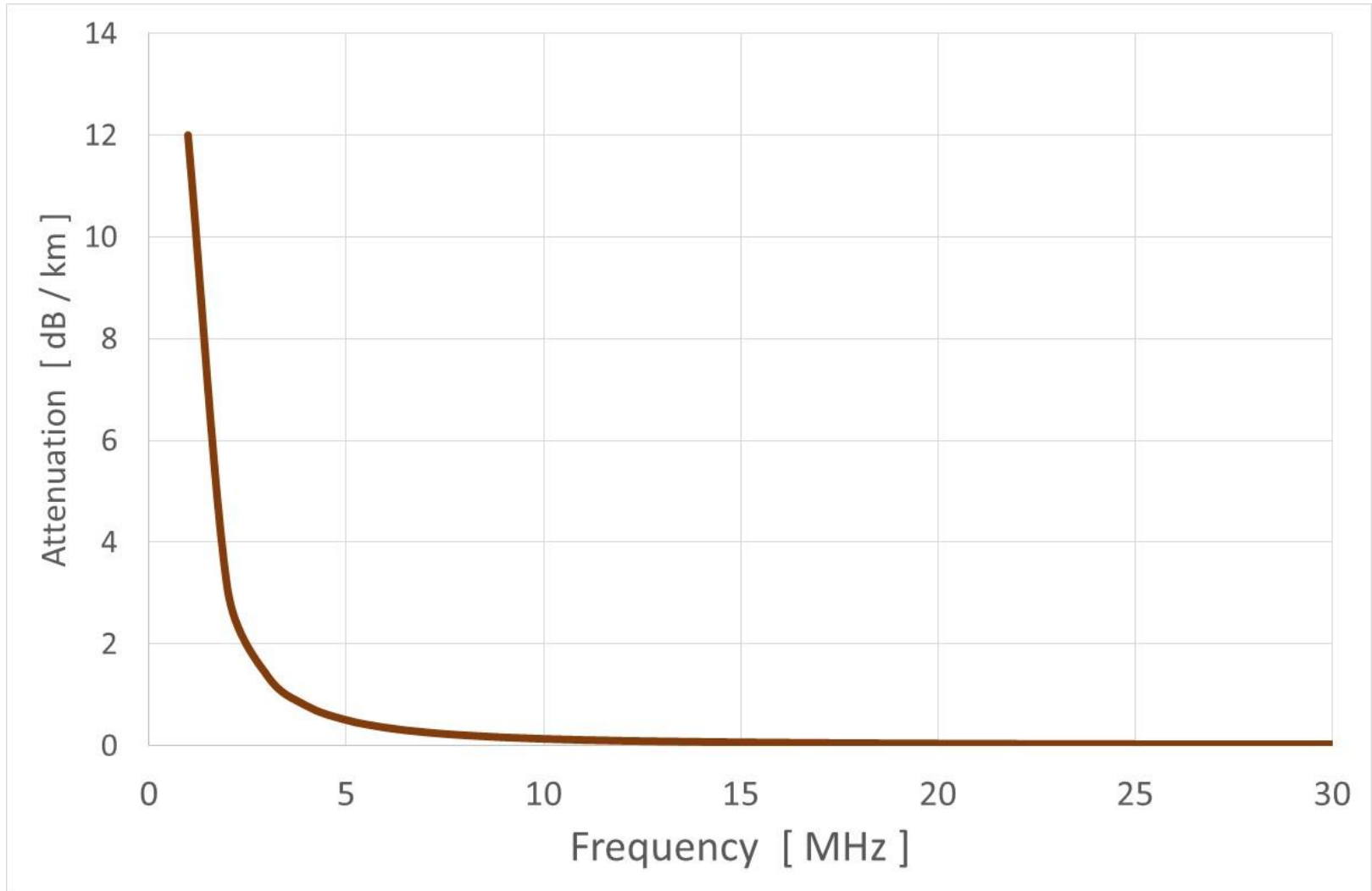
D – layer characteristics

Electron density: $N_e = 10^9 \frac{\text{electrons}}{\text{m}^3}$

Plasma (critical) frequency: $f_p = 8.977 \sqrt{N_e} = 284 \text{kHz}$

Collision rate: $f_e = 1.2 \cdot 10^6 \text{ collisions per second}$

D – layer attenuation

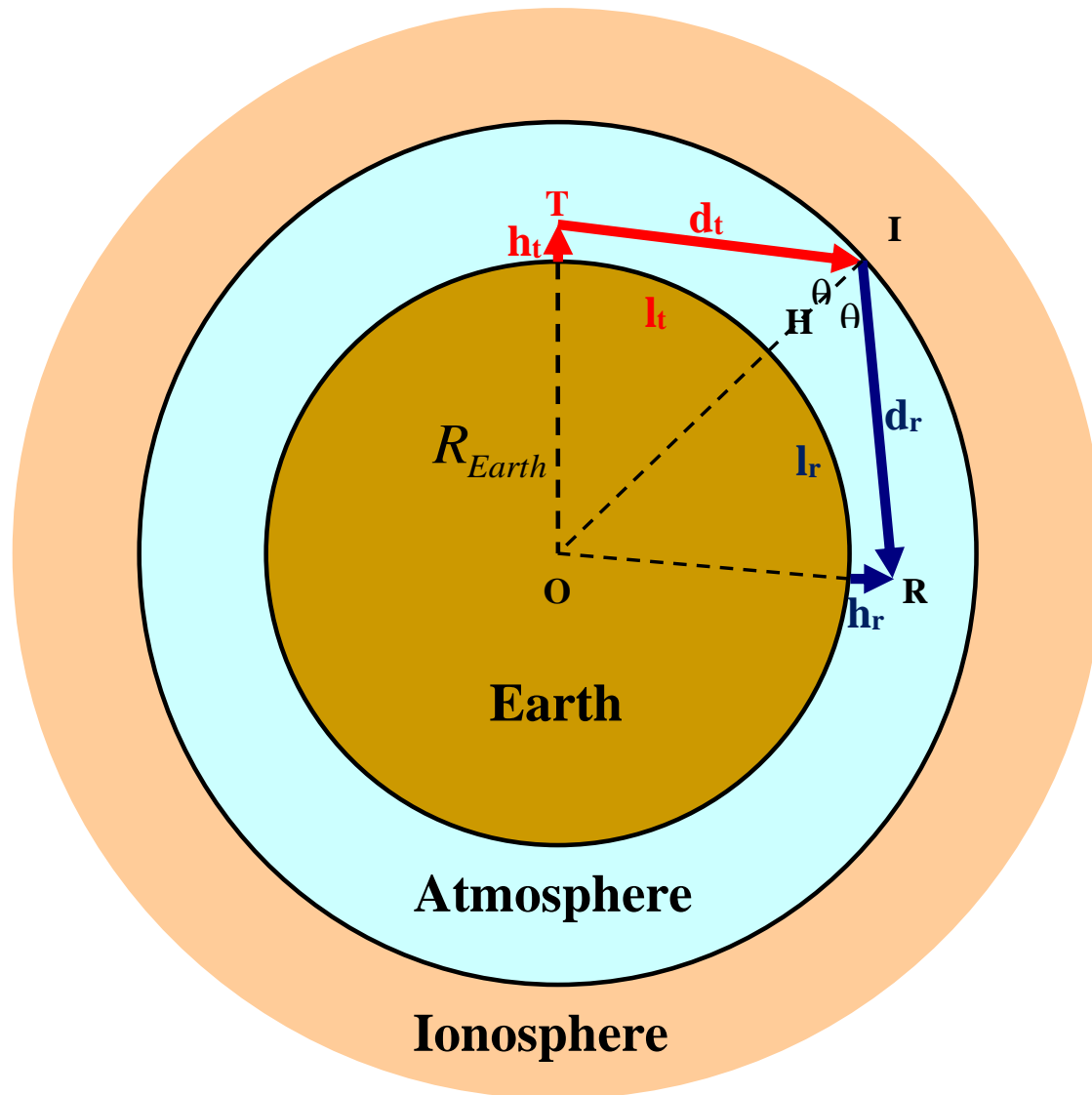


Optimum working frequency (OWF)

Frequency of optimum transmission (FOT):

$$FOT = 0.85 \cdot MUF = 0.85 \cdot \frac{f_p}{\cos(\theta_{\max})}$$

Maximum range



Maximum range

$$(h_t + R_{Earth})^2 = d_t^2 + (H_{Ionosphere} + R_{Earth})^2 - 2d_t R_{Earth} \cos(\theta)$$

$$\frac{d_t}{\sin\left(\frac{l_t}{R_{Earth}}\right)} = \frac{h_t + R_{Earth}}{\sin(\theta)}$$

$$(h_r + R_{Earth})^2 = d_r^2 + (H_{Ionosphere} + R_{Earth})^2 - 2d_r R_{Earth} \cos(\theta)$$

$$\frac{d_r}{\sin\left(\frac{l_r}{R_{Earth}}\right)} = \frac{h_r + R_{Earth}}{\sin(\theta)}$$

$$l = l_t + l_r$$

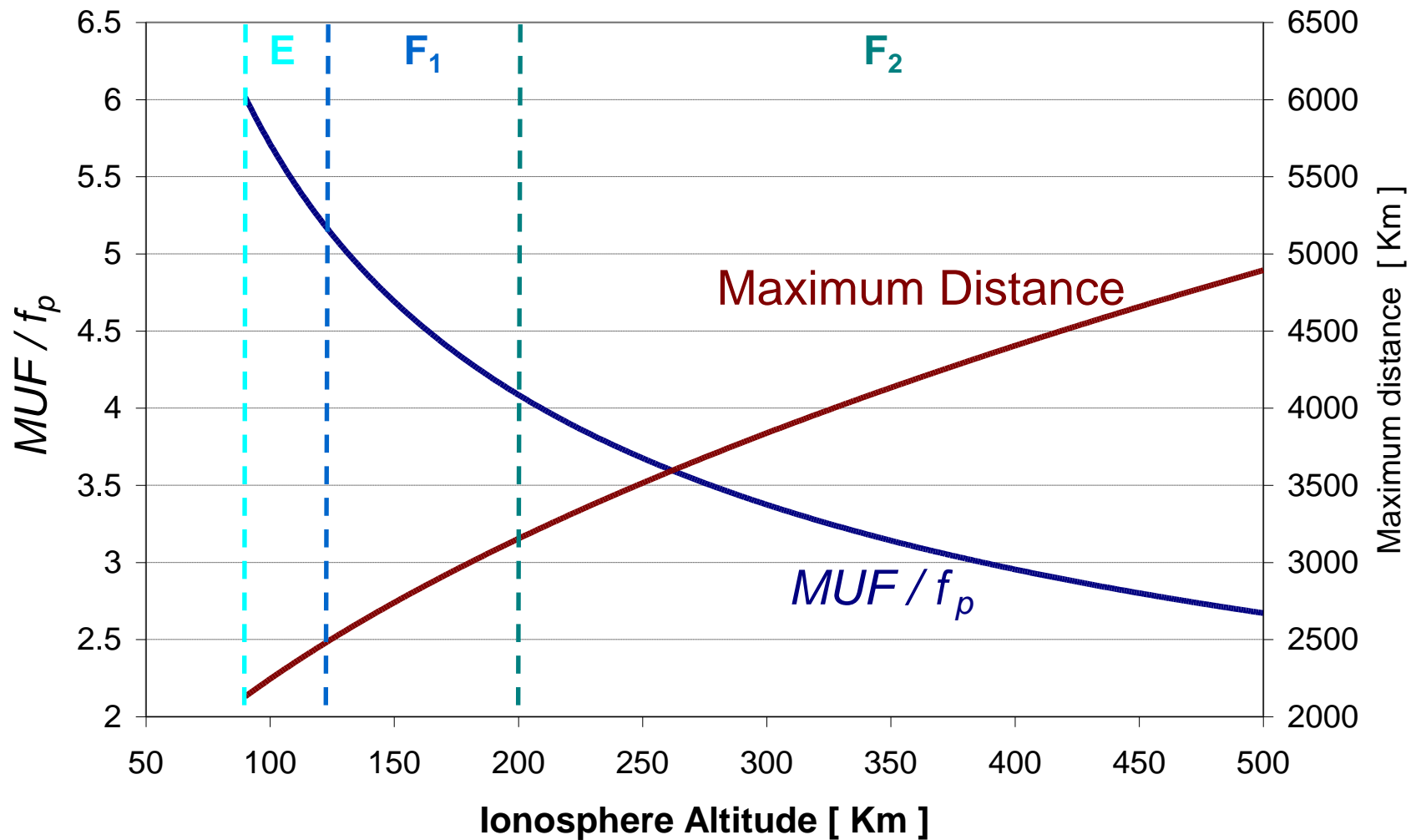
Maximum range

$$h_t = h_r \approx 0$$

$$\sin(\theta_{\max}) = \frac{R_{Earth}}{R_{Earth} + H_{Ionosphere}} \longrightarrow \theta_{\max} = \arcsin\left(\frac{R_{Earth}}{R_{Earth} + H_{Ionosphere}}\right)$$

$$l_{\max} = (\pi - 2\theta_{\max}) \cdot R_{Earth} = \left[\pi - 2 \cdot \underbrace{\arcsin\left(\frac{R_{Earth}}{R_{Earth} + H_{Ionosphere}}\right)}_{\theta_{\max}} \right] \cdot R_{Earth}$$

Maximum range vs. Altitude



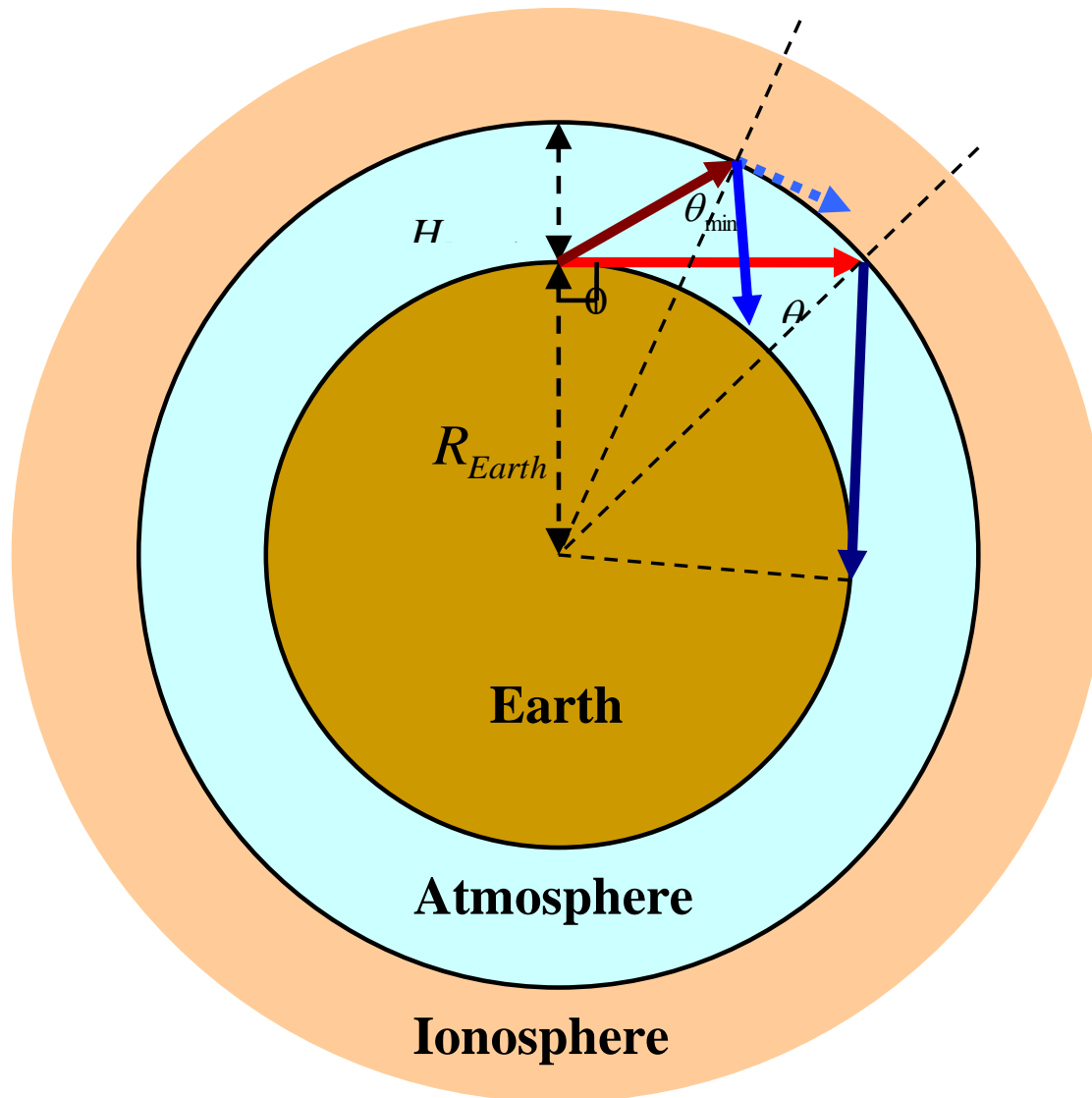
Important notes !

$$f_p \uparrow \longrightarrow MUF = \frac{f_p}{\cos(\theta_{\max})} \uparrow$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow \cos(\theta_{\max}) \uparrow \longrightarrow MUF \downarrow$$

$$H_{\text{Ionosphere}} \uparrow \longrightarrow \sin(\theta_{\max}) \downarrow \longrightarrow \theta_{\max} \downarrow \longrightarrow l_{\max} \uparrow$$

Minimum range



Minimum range

$$\sin(\theta_{\min}) = \sqrt{1 - \left(\frac{f_p}{f}\right)^2} \longrightarrow \theta_{\min} = \arcsin \left[\sqrt{1 - \left(\frac{f_p}{f}\right)^2} \right]$$

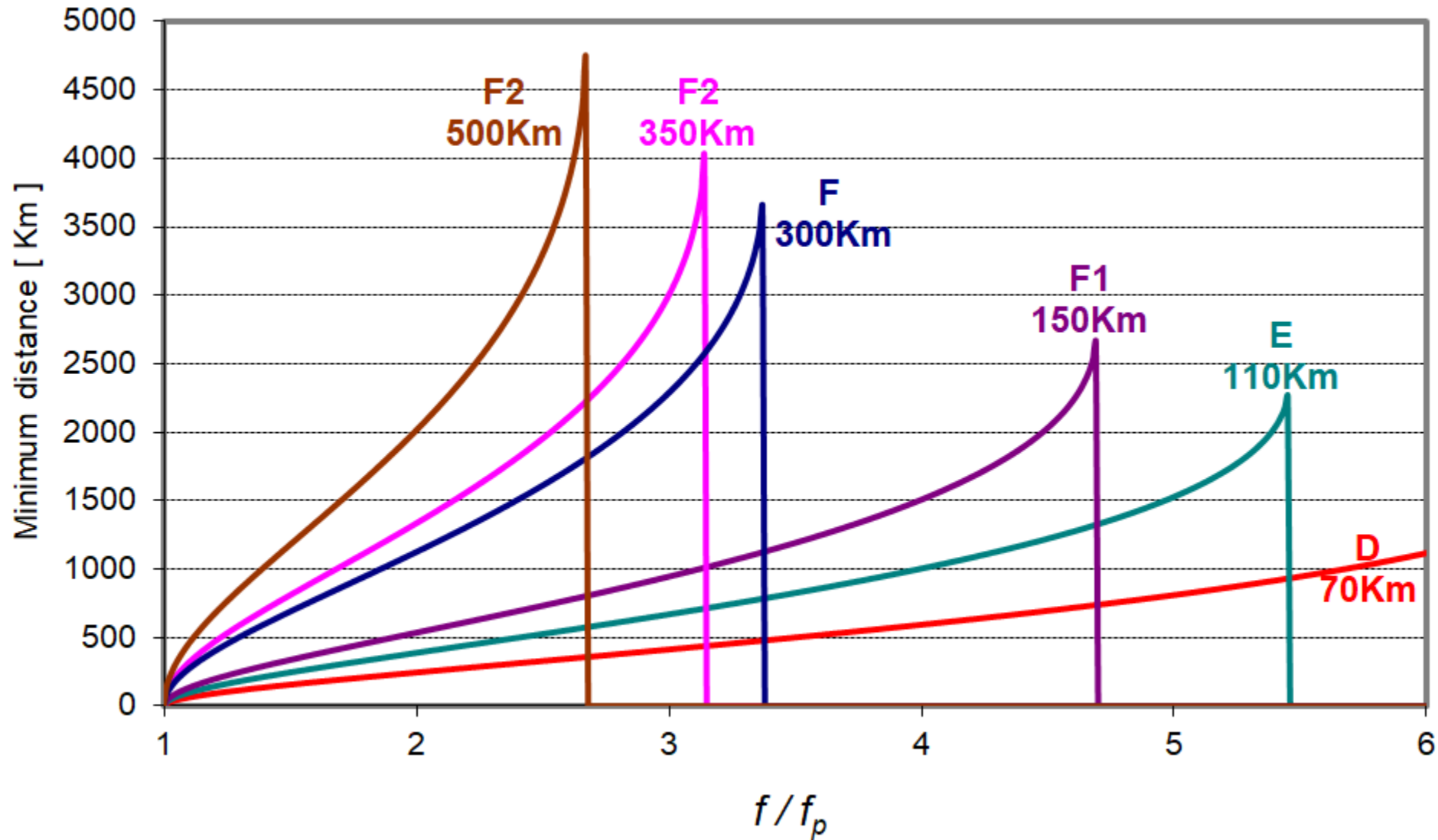
$$\frac{R_{Earth}}{\sin(\theta_{\min})} = \frac{R_{Earth} + H_{Ionosphere}}{\sin(\phi)} \longrightarrow$$

$$\longrightarrow \sin(\phi) = \underbrace{\frac{R_{Earth} + H_{Ionosphere}}{R_{Earth}}}_{1/\sin(\theta_{\max})} \cdot \sin(\theta_{\min}) = \frac{\sin(\theta_{\min})}{\sin(\theta_{\max})}$$

Skip distance

$$\begin{aligned}
 l_{\min} &= 2(\pi - \theta_{\min} - \phi) \cdot R_{Earth} = 2 \cdot \left\{ \pi - \theta_{\min} - \underbrace{\arcsin \left[\frac{\sin(\theta_{\min})}{\sin(\theta_{\max})} \right]}_{\phi} \right\} \cdot R_{Earth} = \\
 &= 2 \cdot \left\{ \underbrace{\pi - \arcsin \left[\sqrt{1 - \left(\frac{f_p}{f} \right)^2} \right]}_{\theta_{\min}} - \underbrace{\arcsin \left[\left(1 + \frac{H_{Ionosphere}}{R_{Earth}} \right) \cdot \sqrt{1 - \left(\frac{f_p}{f} \right)^2} \right]}_{\phi} \right\} \cdot R_{Earth}
 \end{aligned}$$

Minimum range vs. frequency



Important notes !

$$H_{\text{Ionosphere}} \uparrow \longrightarrow MUF \downarrow$$

$$f \uparrow \longrightarrow l_{\text{min}} \uparrow$$

Summary

Time	Layer	Altitude	Electron density	Plasma (critical) frequency	Maximum Usable Frequency	Minimum distance	Maximum distance
		$H_{\text{Ionosphere}}$	N_e	$f_p = 8.977 \sqrt{N_{e_{\text{MAX}}}}$	$MUF = \frac{f_p}{\cos(\theta_{\text{max}})}$	l_{min}	l_{max}
Day	D	70Km	$4 \cdot 10^8$	180KHz	1.22MHz	518Km@500KHz	377Km
Day	E	110Km	$2 \cdot 10^{11}$	4.01MHz	21.9MHz	518Km@7MHz	321Km
Day	F1	180Km	$2 \cdot 10^{11}$	4.01MHz	17.3MHz	530Km@7MHz 1500Km@14MHz	2995Km
Day	F2	325Km	$2 \cdot 10^{12}$	12.7MHz	41.3MHz	304Km@14MHz 900Km@21MHz 1736Km@28MHz	3988Km
Day	F2	250Km	$4 \cdot 10^{12}$	18MHz	66MHz	306Km@21MHz 616Km@28MHz	3514Km
Night	E	100Km	$1.5 \cdot 10^9$	348KHz	1.99MHz		2244Km
Night	F	300Km	$5 \cdot 10^{10}$	2.01MHz	6.77MHz	904Km@3.5MHz	3838Km
Night	F	450Km	$5 \cdot 10^{11}$	2.54MHz	7.11MHz		4657Km

עם ישראל חי !

